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06CS42

Fourth Semester B.E. Degree Examination, June/July 2013
Graph Theory and Combinatorics

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Define isomorphism of graphs. Give an example to show that two graphs need not be isomorphic though they have equal number of edges, equal number of vertices with a given degree. **(08 Marks)**

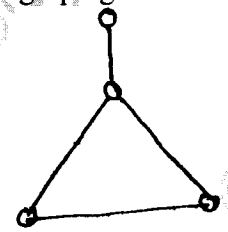
b. Write a note on "Konigsberg-Bridge" problem. **(06 Marks)**

c. Define the following terms with an examples:
i) Spanning subgraph
ii) Complement of a graph
iii) Self complementary graph. **(06 Marks)**

2 a. Show that every simple graph has number of vertices of odd degree is even. **(06 Marks)**

b. Prove that a simple connected graph with n vertices ($n \geq 3$) is Hamiltonian if the degree of every vertex is greater than or equal to n/2. **(06 Marks)**

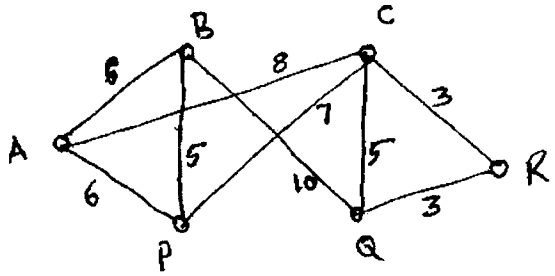
c. Find the chromatic polynomial for the graph given below: **(08 Marks)**



3 a. Obtain a prefix code for the message ROAD IS GOOD using labeled binary tree and hence encode the message. **(08 Marks)**

b. Show that the complete graph K_5 is non planar. **(05 Marks)**

c. Using the Kruskal's algorithm, find a minimal spanning tree of the weighted graph shown below: **(07 Marks)**



4 a. Explain Prims algorithm for finding shortest spanning tree of a weighted graph. **(06 Marks)**

b. Show that in any connected planar graph with n vertices, e edges and f faces e – n + 2 = f. **(08 Marks)**

c. State and prove "Max flow-Min cost" theorem. **(06 Marks)**

PART – B

5 a. Using the principle of inclusion-exclusion, determine the number of positive integers n, where $1 \leq n \leq 100$ and n is not divisible by 2 or 3 or 5. **(08 Marks)**

b. Find the coefficient of x^9y^3 in the expansion of $(2x - 3y)^{12}$. **(06 Marks)**

c. A woman has 11 closed relatives and she wishes to invite 5 of them to dinner. In how many way can she invite them in the following situations:
   i) There is no restriction on the choice.
   ii) Two particular persons will not attend separately.
   iii) Two particular parsons will not attend together. **(06 Marks)**

6 a. Out of 30 students in a hostel, 15 study history 8 study economics and 6 study geography. It is known that 2 student study all three subjects. Show that 7 or more students study none of these subjects. **(06 Marks)**

b. Define ordinary generating function and the exponential generating function. Give one example for each. **(06 Marks)**

c. Find the coefficient of X^{18} in the product $(x + x^2 + x^3 + x^4 + x^5)(x^2 + x^3 + x^4 + \ldots)^5$. **(08 Marks)**

7 a. Define recurrence relation and give two examples. **(04 Marks)**

b. Solve the recurrence relation $a_n - 3a_{n-1} = 5 \times 3^n$, for $n \geq 1$ given that $a_0 = 2$. **(08 Marks)**

c. Determine the sequence generated by each of the following exponential generating function:
   i) $6e^{5x} - 3e^{2x}$
   ii) $e^{2x} - 3x^3 + 5x^2 - 7x$. **(08 Marks)**

8 a. Using generating function, find the number of partition of n = 6. **(07 Marks)**

b. Determine the solution for $a_n = 7a_{n-1}$, where $n \geq 1$, given that $a_2 = 98$. **(07 Marks)**

c. Write the procedure of method of generating functions. **(06 Marks)**

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